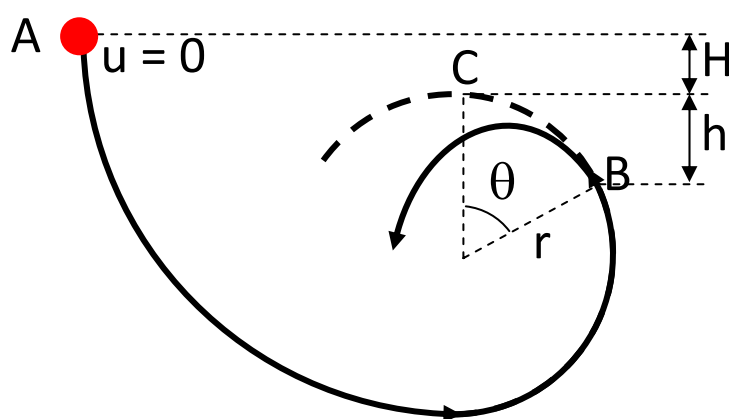


Condition for Interrupted Pendulum Undergoing a Complete Vertical Circular Motion



The bob is forbidden from going outside, but not inside. All frictional forces are neglected. The bob is released at A with initial velocity $u = 0$. By conservation of energy, the bob will attain the speed

$$v_B = \sqrt{2g(h + H)} \quad \dots\dots (1)$$

at B. Suppose at B, the string is still taut, so the net force acting on the bob towards the center is

$$T + mg\cos\theta \quad \dots\dots (2)$$

where T is the tension in the string. If the bob does undergo a circular motion, it must satisfy the condition

$$\text{net radial force} = \frac{mv^2}{r}$$

Hence, by eq (1) and eq (2), the condition for continuing circular motion at B is

$$T + mg\cos\theta = \frac{mv_B^2}{r}, \text{ or}$$

$$T + mg\cos\theta = \frac{2mg(h+H)}{r}. \quad \dots\dots (3)$$

As the bob goes higher, h will be smaller, but $mg\cos\theta$ larger. Therefore, the case

$$mg\cos\theta > \frac{2mg(h+H)}{r}$$

may appear if H is not large enough. If so, eq (3) will no longer be satisfied since the string cannot be compressed, i.e., $T \geq 0$. (c.f. a rigid stick).

Hence, the bob is about to leave the vertical circle when

$$mg\cos\theta = \frac{2mg(h+H)}{r} \quad \dots\dots (4)$$

$$\text{Since } h = r(1 - \cos\theta) \quad \dots\dots (5)$$

So, we get

$$\cos\theta = \frac{2}{3} \left(1 + \frac{H}{r}\right) \quad \dots\dots (6)$$

Eq (6) is used to calculate the angular position for the bob leaving the circle.

- In particular, **if $H = 0$, $\theta = 48.2^\circ$**

If the bob does not leave throughout, $\cos\theta \geq 1$, implying $H \geq \frac{r}{2}$

- So, **the least H for making complete circular motion is $H = \frac{r}{2}$.**

Then, at the top ($h = 0$, $\theta = 0^\circ$), $T = 0$ and the centripetal force required is purely provided by mg .